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Whether or not to run in the rain

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Abstract

The problem of choosing an optimal strategy for moving in the rain has attracted considerable attention among physicists and other scientists. Taking a novel approach, this paper shows, by studying simple shaped bodies, that the answer depends on the shape and orientation of the moving body and on wind direction and intensity. For different body shapes, the best strategy may be different: in some cases, it is best to run as fast as possible, while in some others there is an optimal speed.

(Some figures may appear in colour only in the online journal)

1. Introduction

Is it better to walk or run in the rain? Almost everyone has at least once faced this question; many discussions on this topic can be found on the web, with various conclusions. The problem has also been addressed by some physicists, mathematicians, engineers and meteorologists. Some studies [1] consider vertically falling rain. Others [2, 3] take into account the possibility of wind with the same direction of motion. Others still [4–8] also consider a cross (i.e. perpendicular to the path) component of the wind. Since the human body has a very complex shape, a much simpler form, usually a parallelepiped, has been considered, with the implicit or explicit assumption that shape is not a crucial factor, and that results obtained for a parallelepiped can be quite easily generalized to bodies of any shape.

Qualitatively speaking, the results found up to now can be summarized as follows.

- For rain falling vertically, the best strategy is to run as quickly as possible. The same is also true for motion into the wind.
- For motion downwind, there may be an optimal speed, which equals the component along the direction of motion of the wind velocity. This happens only if the ratio between the cross-section of the body perpendicular to the motion and the horizontal one is large enough; otherwise, the best choice is again to run at the maximum speed one can reach.

In essence, in the literature so far published on this subject there are two main conclusions: for the existence of an optimal speed the wind must come from behind and the optimal speed, when it exists, equals the component of the wind velocity along the path. In this paper, we shall show that the answer to the question actually depends on many factors, mainly on the shape of the body and its orientation. Even though a tailwind is a favourable condition for the existence of an optimal speed, in some cases we can have one even with a headwind, and in general its value is not equal to that found in previous studies.

The paper is addressed mainly to undergraduate students and their teachers, as well as to anyone intrigued by the problem. The subject requires good geometrical visualization, a good command of vector calculus and involves many concepts from various fields; as such, it could be useful in physics courses at university level.

2. Definition of the problem

The question stated at the very beginning of this paper is too vague, and should be better defined.

Let us consider a fixed track that has to be covered in the rain. We shall assume the following.

(1) The ground is horizontal.
(2) The path is rectilinear.
(3) The rain is uniform, at least along the path.
(4) The rain is constant, at least for the time needed to cover the path.
(5) The body motion is rigid and translatory.

This last assumption is clearly a quite crude approximation of a human body, but it is necessary to make the problem manageable. The conditions (2), (3) and (4) are not as restrictive as they may look at first sight: if they are not satisfied on the whole path, we can always split it into shorter parts.

3. Introduction of conceptual tools

Let us suppose that we have to go straight from point A to point B. We choose a Cartesian reference system with the x axis oriented from A to B, and the z axis vertical. Let $\mathbf{u}$ be our velocity; in this frame, $\mathbf{u} = (u, 0, 0)$, with $u > 0$. The problem is to find the value of u, if any, that will minimize the mass of water impinging on our body along the path.

Let $\mathbf{v} = (v_x, v_y, v_z)$ be the rain velocity. The vertical component, v_z, depends on the drop size. We shall assume that drops fall at their terminal speed, so that their horizontal velocity equals that of the wind (we do not restrict ourselves to a wind along the x direction, but we assume a horizontal wind). The sign of v_y simply tells us if we get wet on our left or right side. When v_x is positive, we shall speak of a tailwind, otherwise of a headwind, irrespective of the presence of a cross component v_y.

The plane identified by $\mathbf{u}$ and $\mathbf{v}$ plays an important role in the problem: we shall call it π; an axis perpendicular to this plane will be denoted by ζ.

Let us call ρ the ratio between the mass of water drops that are found, at some instant, within a given volume, and the volume itself (note that ρ is *not* the water density). We can now define a vector $\mathbf{j_0}$ that we shall call rain density:

$$\mathbf{j_0} = \rho\mathbf{v}. \tag{1}$$

It is easy to recognize the analogy between ρ and charge density, and between $\mathbf{j_0}$ and the current density vector in electromagnetism. Our approach will freely exploit conceptual tools taken from this field. Assumptions (3) and (4) imply ρ and $\mathbf{v}$—and consequently $\mathbf{j_0}$—to be uniform and constant.

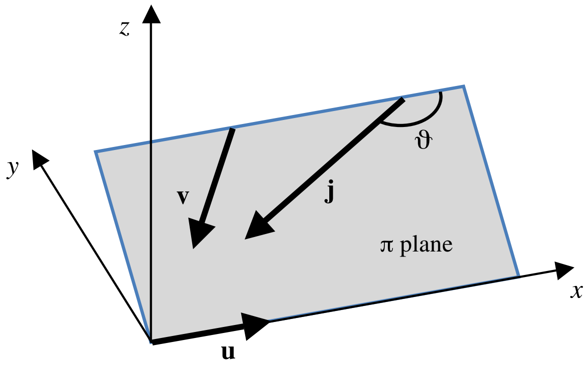

Figure 1. The relative orientation of **v**, **u**, **j** and the plane π, which contains all of them.

When moving, we shall perceive an apparent rain density **j**, which will differ from $\mathbf{j_0}$:

$$\mathbf{j}(u) = \rho(\mathbf{v} - \mathbf{u}) = \rho(v_x - u, v_y, v_z). \tag{2}$$

The **j** vector changes, in intensity and direction, with u. Its direction rotates, when u increases, in the plane π, drifting away from the positive x direction; we can easily see this effect on train or car windows. In astronomy, this effect is known as *stellar aberration*. Sailors know well that the direction from which the apparent wind *comes* approaches the bow as boat speed increases. We can state this concept more precisely by saying that the angle θ between **j** and **u** increases with u, approaching 180°. It is easy to show that

$$\tan\theta = \frac{\sqrt{v_y^2 + v_z^2}}{v_x - u}. \tag{3}$$

We can note that $\mathbf{j_0}$ is the value of **j** corresponding to $u = 0$; we shall call it the *initial* value of **j**. Similarly, the angle between $\mathbf{j_0}$ and **u** will be denoted by θ_0.

Figure 1 shows the relative orientation of **v**, **u**, **j**, and the plane π.

Using once more the analogy with electromagnetism, it is natural at this point to introduce the concept of rain flux. If we adopted without any change the flux definition for a vector field, we should define rain flux as the surface integral of the **j** field:

$$\Phi(u) = \oint_S \mathbf{j} \cdot \mathrm{d}\mathbf{A}, \tag{4}$$

where the integration is over the whole surface of the body. But we know that, in the absence of field sources or wells, the integral over a closed surface vanishes: thus the integral (4) must be modified for our purposes.

A first change is to consider the modulus of the dot product $\mathbf{j} \cdot \mathrm{d}\mathbf{A}$. But this is not satisfactory, as every field line contributes twice to the flux: when it enters the body and when it leaves. So we shall restrict the integration to what we shall call the 'wet surface' of our body, S_w, that is the surface where field lines enter the body. If the body has a complex shape, so that there are some field lines that enter, go out and enter again, the wet surface obviously includes only those parts where field lines enter the body for the first time. We then define rain flux as:

$$\Phi(u) = \int_{S_w} |\mathbf{j} \cdot \mathrm{d}\mathbf{A}|. \tag{5}$$

Note that, while S is a closed surface, S_w is always an open one, as the different notation of the integral remind us. We shall assume that it maintains the same orientation as S (which, by convention, is oriented from inside to outside).

Rain flux is the ratio between the mass of water impinging on the body during a given time interval and the time interval; its SI measure unit will be kg s^{-1}. It depends on the body speed for two reasons, because **j** changes with u and because, due to the rotation of **j**, the surface of integration may change.

4. Formalisation and general discussion of the problem

We can now, at least in theory, calculate the water mass, m, hitting the body during motion in the rain. The time interval, Δt, needed to travel a distance L at a constant speed u is $\Delta t = L/u$, so that m is given by

$$m(u) = \Phi\,\Delta t = \frac{L}{u}\int_{S_w} |\mathbf{j}\cdot \mathrm{d}\mathbf{A}|. \tag{6}$$

While this equation is adequate to study the behaviour of solid bodies enclosed in plane surfaces (e.g. a parallelepiped), in the presence of curved surfaces it is in general more convenient to write it in a different way, using the following theorem.

The absolute value of the flux of a uniform vector field over a surface, S, such that each field line intersects the surface at most once, is given by the product of the field intensity by the area of the projection, S_{pr}, of the surface S on a plane orthogonal to the field.

The proof follows from the definition of the flux, or Gauss's theorem. Using this result, the water mass can be written as

$$m(u) = \frac{L}{u}|\mathbf{j}(u)|S_{pr}(u). \tag{7}$$

The time L/u clearly decreases monotonically with u. The projected wet surface S_{pr} depends on the shape of the body and on its orientation with respect to the rain and the direction of motion—we cannot say anything in general about its dependence on u. We note also that the projection of the wet surface exactly overlaps with the projection of the whole body surface.

The absolute value of **j** (see equation (2)) increases monotonically with u if $v_x < 0$. We know, when travelling in the rain, that if we speed up, the intensity of the rain seems to increase. On the contrary, if $v_x > 0$, the apparent rain density has a minimum for $u = v_x$.

The ratio $|\mathbf{j}|/(u)$ does not depend on the body: it is therefore connected with some general features of the problem and it is worthwhile to study its dependence on u. Let us write it in a more explicit form:

$$\frac{|\mathbf{j}|}{u} = \frac{\rho\sqrt{(v_x-u)^2+v_y^2+v_z^2}}{u} = \rho\sqrt{\left(\frac{v}{u}\right)^2 - 2\frac{v_x}{u}+1} = \rho\sqrt{\left(\frac{v}{u}\right)^2 - 2\frac{v}{u}\cos\theta_0+1}. \tag{8}$$

It is remarkable that this function, while monotonically decreasing for $v_x < 0$, always has a minimum when $v_x > 0$, that is for

$$u = \frac{v^2}{v_x} = \frac{v}{\cos\theta_0}. \tag{9}$$

A plot of $|\mathbf{j}|/(u)$ versus u/v for some values of θ_0 is shown in figure 2. We can see that for $\theta_0 = 30^\circ$ the function has a well pronounced minimum, while for 60° the minimum is already hard to distinguish.

This approach automatically solves the problem for all situations where the projected wet surface does not depend on u. Of course, the simplest example one can consider is a sphere,

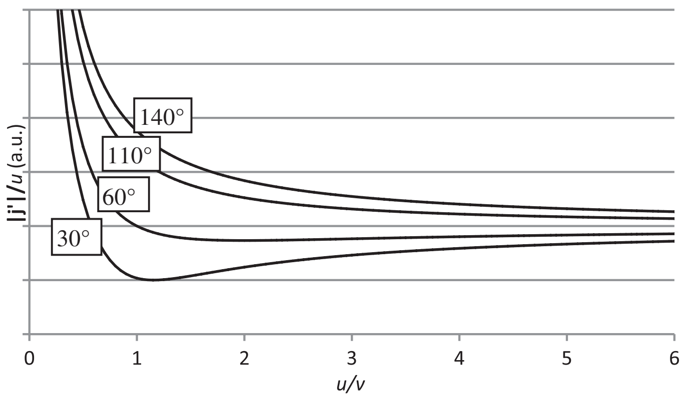

Figure 2. Plot of the ratio $|\mathbf{j}|/(\rho u)$ (arbitrary units) versus u/v for the following values of the angle θ_0 between $\mathbf{u}$ and $\mathbf{j}_0$: 30° and 60° (tailwind), 110° and 140° (headwind).

for which the projected wet surface is always πR^2, R being the sphere's radius. But this is not the only case: as we have seen, when u changes, $\mathbf{j}$ rotates in a plane perpendicular to the ζ axis, so every body with a symmetry axis along this direction is in the same situation.

In conclusion, we can say that in all the cases where the projected wet surface does not depend on u.

- An optimal speed exists, and its value is always given by equation (9), *irrespective of the body shape* (cylinder, cone, ellipsoid ...), under the only condition that the wind comes from behind. We note that the u value (9) is always greater than v_x. Thus the optimal speed, when it exists, is not always equal to v_x, as found by previous authors.
- The optimal speed depends on v, and then on the drop size (since v obviously depends on v_z).
- An optimal speed exists irrespective of the intensity of the wind, provided that it has a tail component.

Of course, in general S_{pr} does depend on u, and its effect on $m(u)$ may be very different from one case to another. We can say that it is more relevant for bodies with a high degree of asymmetry with respect to rotations around the ζ axis. We can be more precise only for some particular, simple situations.

5. A parallelepiped and a plane surface

As said in the introduction, previous papers (with the exception of Ehrmann and Blachowicz [2], who considered a vertical cylinder) have so far considered a parallelepiped with edges parallel to Cartesian axes (we shall call it a 'vertical' parallelepiped) and a plane surface. Let us now direct our attention to these two cases.

5.1. A vertical parallelepiped

The case of a vertical parallelepiped has been discussed by several authors, but we shall restate the results briefly here with our notation for the sake of convenience.

The rain flux results:

$$\Phi(u) = \rho(S_x|v_x - u| + S_y|v_y| + S_z|v_z|). \tag{10}$$

Here S_x denotes the area of each face perpendicular to x axis, and so on. As a consequence, the mass of the intercepted water is:

$$m(u) = \frac{\rho L}{u}(S_x|v_x - u| + S_y|v_y| + S_z|v_z|). \tag{11}$$

Depending on the value of v_x, we must consider the following cases.

- $v_x \leqslant 0$: *headwind or no wind.*
  In this case $|v_x - u| = |v_x| + u$. Then:
  $$m(u) = \rho L\left(S_x + \frac{S_x|v_x| + S_y|v_y| + S_z|v_z|}{u}\right). \tag{12}$$
  This function decreases monotonically, so the best strategy is to move as quickly as possible.
- $v_x > 0$: *tailwind.*
  At 'low speed' ($u < v_x$), $|v_x - u| = v_x - u$, so that
  $$m(u) = \rho L\left(-S_x + \frac{S_x|v_x| + S_y|v_y| + S_z|v_z|}{u}\right). \tag{13}$$
  This function, too, decreases monotonically, so it is never convenient to move at a speed lower than v_x. But what about a speed higher than v_x?
  At 'high speed' ($u \geqslant v_x$), $|v_x - u| = u - v_x$, then:
  $$m(u) = \rho L\left(S_x + \frac{-S_x|v_x| + S_y|v_y| + S_z|v_z|}{u}\right) \tag{14}$$
  and we must further differentiate between two subcases, depending on the sign of the expression:
  $$- S_x|v_x| + S_y|v_y| + S_z|v_z|. \tag{15}$$
  When this expression is strictly positive, $m(u)$ also decreases for $u \geqslant v_x$, so it is convenient, once more, to move at a maximum speed. But when, on the contrary, it is negative, $m(u)$ increases for $u \geqslant v_x$, an optimal speed does exist and is always:
  $$u_{\text{opt}} = v_x. \tag{16}$$
  We can observe that in this case the optimal speed depends only on the wind speed and not on drop size. We note also that, when expression (15) vanishes, the mass of water intercepted by the body is equal to the water found in the volume LS_x and is independent of u, provided only that $u > v_x$.

It is worthwhile to write down the condition that (15) be negative in two different ways:

$$S_x \; > \; \frac{S_x|v_x| + S_y|v_y}{v_x}, \tag{17a}$$

$$v_x \; > \; \frac{S_x|v_x| + S_y|v_y}{S_x}. \tag{17b}$$

Equation (17a) tells us that, for a given $\mathbf{v}$, an optimal speed exists only for parallelepipeds with a large S_x; (17b) tells that, for a given parallelepiped, an optimal speed exists only if v_x is large enough.

5.2. A generalisation?

For a parallelepiped, the rain flux is the sum of three terms, each relative to a component of $\mathbf{j}$:

$$\Phi = |j_x|S_x + |j_y|S_y + |j_z|S_z. \tag{18}$$

It is very tempting, at this point, to generalize this result to a body of any shape, by simply defining S_x to be the projected area of the body on a plane normal to x axis, and so on, and this seems to be the line followed by previous papers. However, even though very appealing, this idea is not correct, because the presence of the modulus in definition (5) does not permit splitting of the sum of the component products.

We have already seen a simple example where equation (18) is not valid. In the case of a sphere, according to this formula the flux should be $(|j_x| + |j_y| + |j_z|)\pi R^2$, while it turns out to be $|\mathbf{j}|\pi R^2$. Below we shall see some more examples.

5.3. A plane surface

In order to develop a deeper understanding of the physics of the problem, let us consider a plane-oriented surface and $\mathbf{A}$ to be the vector area associated with it. We need to consider two more quantities: the angle, φ, between $\mathbf{A}$ and $\mathbf{j}$, whose initial value (the angle between $\mathbf{A}$ and $\mathbf{j}_0$) will be denoted by φ_0, and the angle, ψ, between $\mathbf{A}$ and $\mathbf{u}$. Without any loss of generality, we can always choose the orientation of $\mathbf{A}$ in such a way that A_x is positive. Equation (6) then gives:

$$m(u) = \frac{L}{u}|\mathbf{j} \cdot \mathbf{A}| = \frac{\rho L}{u}|(\mathbf{v} - \mathbf{u}) \cdot \mathbf{A}| = \rho L\left|\frac{\mathbf{v} \cdot \mathbf{A}}{u} - A_x\right|. \tag{19}$$

From this relation we see the following.

- When $\mathbf{v} \cdot \mathbf{A} = 0$, that is when the surface is parallel to the rain, m does not depend on u, so the speed does not matter at all. The surface simply collects all the water contained in the volume LA_x. In particular, when $A_x = 0$, that is when the surface is in the π plane, it remains dry.
- When $\mathbf{v} \cdot \mathbf{A} \neq 0$ and $A_x = 0$, the way to minimize m is to let u be as high as possible.
- Lastly, when $\mathbf{v} \cdot \mathbf{A}$ and A_x are both non-zero, the only relevant parameter is the angle φ_0, which determines the sign of the dot product $\mathbf{v} \cdot \mathbf{A}$. Then we have that when φ_0 is acute, it is always possible to choose a speed such that the surface does not get wet at all. This optimal speed is clearly
$$u_{\text{opt}} = \frac{\mathbf{v} \cdot \mathbf{A}}{A_x} = v\frac{\cos\varphi_0}{\cos\psi_0}. \tag{20}$$
When, on the contrary, φ_0 is obtuse, we see from equation (19) that $m(u)$ decreases monotonically, so the best strategy is again to move as quickly as possible (the surface should move in the opposite direction in order to avoid getting wet).

It can be useful to plot expression (19), which is essentially the modulus of a vertically shifted hyperbola branch, for the case of an acute and an obtuse value of φ_0 (see figure 3).

In conclusion, we can say that for a plane surface the choice of the best strategy depends uniquely, for a given rain velocity, on the orientation of the surface with respect to the rain (which determines the sign of the dot product $\mathbf{v} \cdot \mathbf{A}$) and on the direction of motion (which determines if A_x vanishes or not).

Compared to the case of a vertical parallelepiped, for a plane surface the following differences can be highlighted.

- When $m(u)$ has a minimum, it is always zero, that is, the surface remains dry.
- An optimal speed can also exist with a headwind.
- There is no condition on v_x; an optimal speed can also exist with a light wind.
- The optimal speed can be either lower or higher than v_x.
- The value of u_{opt} also depends on the modulus of the rain velocity, and then on the drop size.

The condition that the angle φ_0 is acute has a simple physical meaning: given our convention to orient $\mathbf{A}$ so that $A_x > 0$ ($\mathbf{A}$ 'points ahead'), it simply means that *the rain wets the rear face* of the surface. In this case, increasing u, the rotation of $\mathbf{j}$ will ultimately

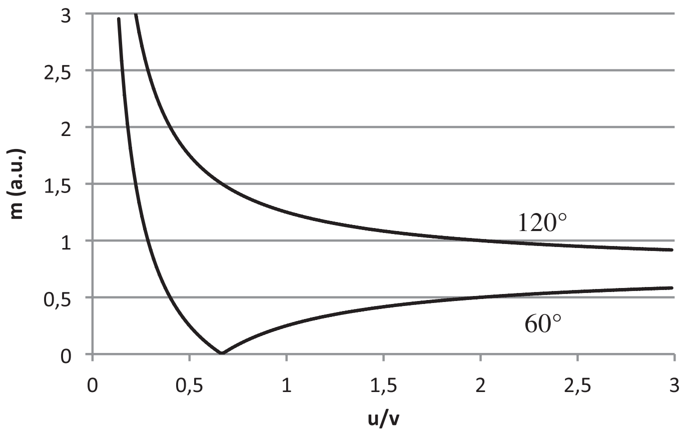

Figure 3. Plot of m (arbitrary units) versus u/v for a plane surface in case of an acute (60°) and obtuse (120°) angle φ_0 between **v** and **A**.

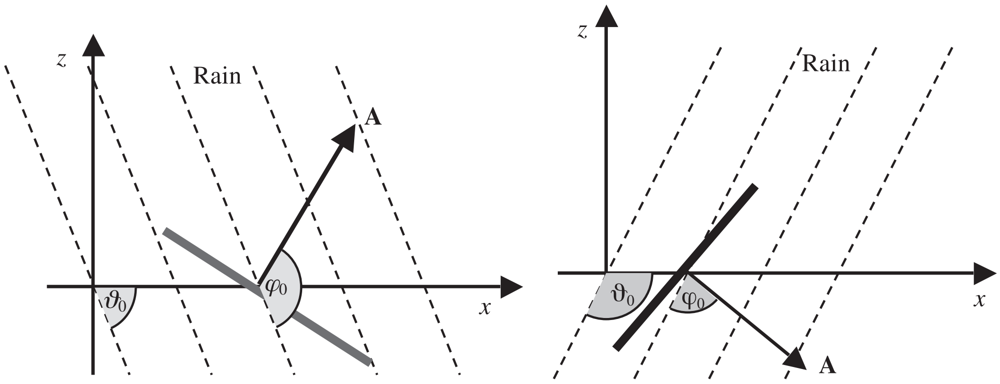

Figure 4. A plane surface parallel to the y axis moves in the rain, in the absence of a lateral wind. (Left) We have a tailwind and the angle φ_0 between the rain and **A** is obtuse (the rain wets the front face of the surface). Increasing u, **j** rotates clockwise and never becomes parallel to the surface, so $m(u)$ does not have a minimum. (Right) We have a headwind and the angle φ_0 is acute (the rain wets the rear face). When **j** rotates, there is a particular (optimal) speed for which it becomes parallel to the surface.

lead to wetting of the front face, and so there will be a particular value of u for which the rain will be parallel to the surface.

Now we can understand that the condition of having a tailwind is neither necessary nor sufficient to have a minimum in the function $m(u)$. Figure 4 explains this point for the cases of $v_y = 0$ (no cross component of the wind) and $A_y = 0$ (surface parallel to the y axis).

From the discussion of this point—which concerns a very simple geometrical situation—we can learn how complex the problem is and how we must pay attention to such aspects as the orientation with respect to the direction of motion and of the rain.

6. From a plane surface to a solid body

Of course, a solid body is not a plane surface, it has several surfaces with different orientations, so it is not surprising that results found in the two cases are different. For example, there is no

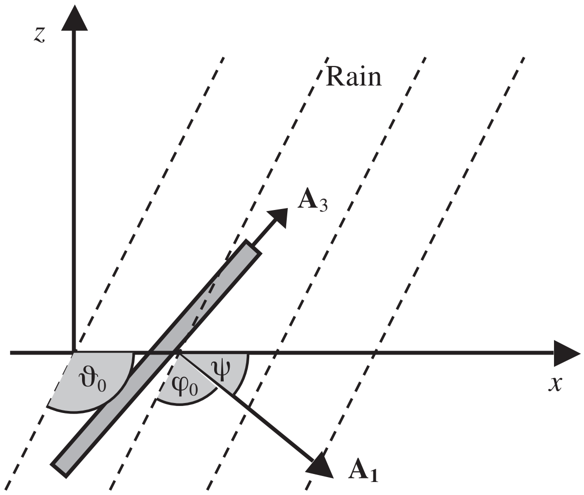

Figure 5. A parallelepiped with two opposite faces parallel to the *xz* plane. By analogy with the previously considered situation, we call ψ the angle between $\mathbf{A}_1$ and the *x* axis. We choose $\mathbf{A}_1$ and $\mathbf{A}_3$ so that their *x* component is positive.

way for a solid body to remain dry in the rain. However, the 'transition' from a plane surface to a solid body, for physical reasons, must be smooth.

Let us consider a parallelepiped with a generic orientation. First of all we observe that in a parallelepiped each pair of opposite faces behaves in the rain like one plane surface, so a parallelepiped, for our discussion, is equivalent to three mutually perpendicular rectangles. If we associate a vector area with each pair we get three mutually orthogonal vectors, that we shall call $\mathbf{A}_1$, $\mathbf{A}_2$, and $\mathbf{A}_3$, and that we shall orient, as before, in such a way that their x component is positive.

The equation that gives the water mass impinging on the parallelepiped is a simple generalisation of equation (19), which reduces to (11) for a vertical parallelepiped:

$$m(u) = \rho L\left(\left|\frac{\mathbf{v}\cdot\mathbf{A}_1}{u} - A_{1x}\right| + \left|\frac{\mathbf{v}\cdot\mathbf{A}_2}{u} - A_{2x}\right| + \left|\frac{\mathbf{v}\cdot\mathbf{A}_3}{u} - A_{3x}\right|\right). \qquad (21)$$

This is the sum of three functions, each of which can have one of the two trends shown in figure 2. The study of this function is a very complex problem, so we shall focus our attention on a slightly simpler situation, that is an analogue of the 'bi-dimensional' case considered in figure 3: a parallelepiped with two faces perpendicular to the y axis (let us call $\mathbf{A}_2$ the vector area associated with this pair) and no cross component of the wind ($v_y = 0$) (see figure 5).

In this situation there is no flux on the faces perpendicular to the y axis. In order to shed some light on the transition from a plane surface to a solid body, we may plot function (21) for increasing values of the ratio A_3/A_1 (see figure 6).

The plot has been obtained with the following parameters: $\theta_0 = 125^\circ$ (headwind) and $\psi = 50^\circ$.

We can see how the thickness affects $m(u)$: it gradually makes the minimum shallower, and ultimately destroys it. The reason is clear; increasing the thickness has the effect of reducing the asymmetry of the projected surface with respect to rotations in the π plane (in this case, the xz plane). At the same time, it can be seen from this plot that *there are cases where, even for a solid body, a minimum exists with a headwind, and where the optimal speed is lower than* v_x; some of the differences between a plane surface and a parallelepiped, described in section 5.3, are just due to the orientation of the solid body.

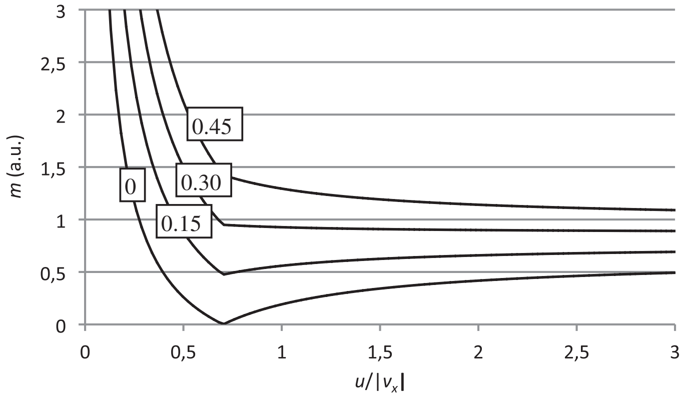

Figure 6. Plot of m (au) versus $u/|v_x|$ for a parallelepiped with $\psi = 50°$ and rain falling at an angle $\theta_0 = 125°$ (headwind), for various values of the ratio A_3/A_1: 0, 0.15, 0.30, 0.45.

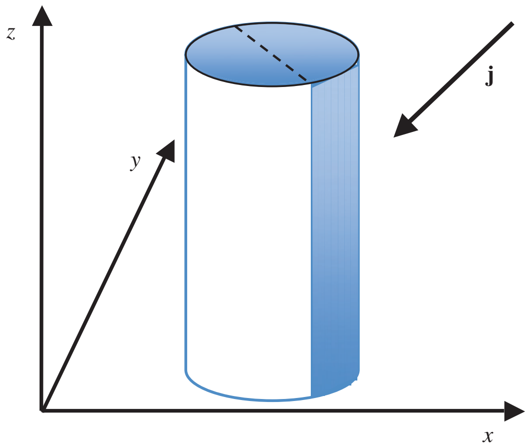

Figure 7. A vertical cylinder. The wet surface is highlighted.

7. A cylinder

7.1. Vertical cylinder

Let us now consider a cylinder oriented vertically. In this case the wet surface S_w is constituted by the upper base and by one half of the lateral surface. To be more precise, it is one of the two halves defined by a plane passing through the cylinder axis and perpendicular to the horizontal component of **j**, which we will denote j_{xy}. The situation is shown in figure 7.

On the upper base, the only contribution to the flux comes from j_z. On the lateral surface, on the other hand, only j_{xy} contributes. With regard to this latter term, it is evident that the associated projected surface S_{pr} is a rectangle with base $2R$ and height H, R and H being, respectively, the radius and height of the cylinder. Then the rain flux is:

$$\Phi(u) = \rho\left(\pi R^2|v_z| + 2RH\sqrt{(v_x - u)^2 + v_y^2}\right). \tag{22}$$

Consequently, the water mass is:

$$m(u) = \rho L R^2\left[\pi\frac{|v_z|}{u} + 2\frac{H}{R}\sqrt{\left(\frac{v_x}{u}\right)^2 - 2\left(\frac{v_x}{u}\right) + 1 + v_y^2}\right]. \tag{23}$$

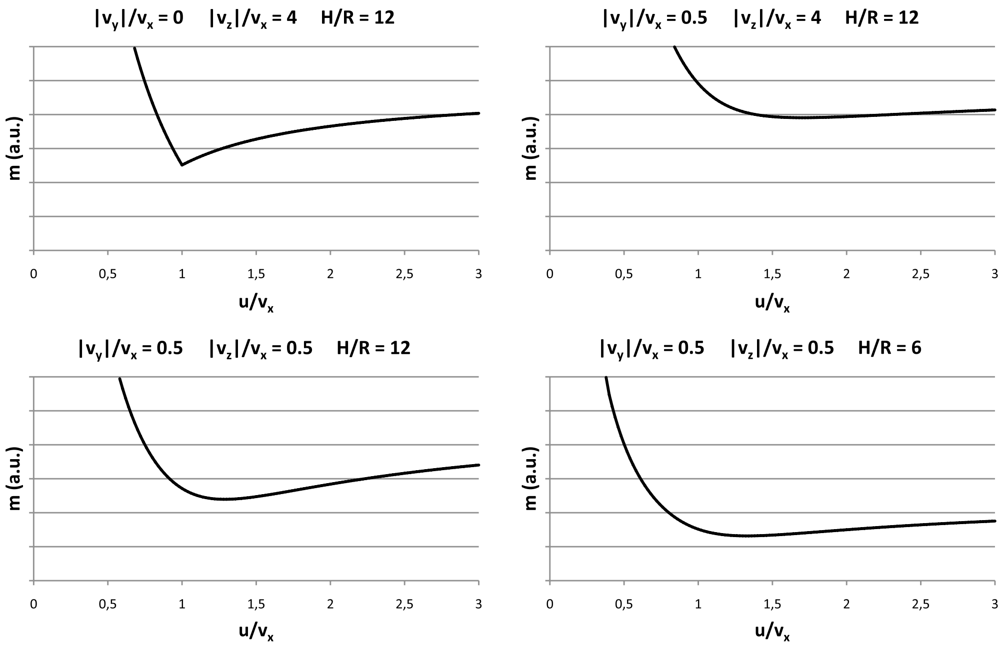

Figure 8. Qualitative plots of m (arbitrary units) versus u/v_x for a vertical cylinder for various sets of parameters. The upper left plot is identical to what can be obtained for a parallelepiped.

This expression is functionally different from that obtained for a parallelepiped, showing one more case where equation (18) is not correct. Without performing a complete study of this function, we can obtain some information from it.

First of all, we may note that for $v_x < 0$ this function is monotonically decreasing; a minimum may exist only with a tailwind.

In case that $v_y = 0$, the cylinder behaves just like a parallelepiped with $A_x = 2RH$ and $A_z = \pi R^2$. In this particular case, then, equation (18) may be applied. A vertical cylinder was considered by Ehrmann and Blachowicz [2] who, however, did not take into account a cross component of the wind and so missed the difference with a parallelepiped.

On the other hand, if $v_y \neq 0$, we can plot the function (23) for various sets of parameters in order to get some idea of its general behaviour (see figure 8).

Looking at these plots, we could conjecture the following.

- Increasing the ratios $|v_y|/v_x$ or $|v_z|/v_x$ has the effect of making the minimum smoother and of shifting the optimal speed towards higher values. If these ratios are too high, a minimum may not exist at all.
- The same effects can be obtained by decreasing the ratio H/R. If this ratio is too low, a minimum may not exist at all.

8. Conclusion

8.1. Results

Starting from very general assumptions, an equation that describes how to calculate the mass of the water received by the body has been derived (we wrote it in two different ways: equations (6) and (7)). We have solved or studied this equation for a plane surface and for bodies with a simple shape. For a plane surface, we have found the following.

- An optimal speed exists subject to the sole condition that the rain wets the rear face of the surface, irrespective of the intensity of the wind and of the sign of v_x.
- The optimal speed is $u_{\text{opt}} = \mathbf{v} \cdot \mathbf{A}/A_x$, so it depends on $|\mathbf{v}|$, and then on the drop size.
- When moving at this optimal speed, the surface does not get wet.
- The optimal speed can have any value.

For solid bodies, we have solved the equation in all cases where the area of the wet surface does not depend on u, that is when the body has a symmetry axis and this is perpendicular to the plane identified by **v** and **u**. In these cases, an optimal speed exists, provided that the wind has a component from behind; its value is $u_{\text{opt}} = v/\cos\theta_0$, where θ_0 is the angle between **v** and **u**, so in this case it also depends on the drop size.

The case of a vertical parallelepiped had been already studied and solved by previous authors. We have extended this study by considering a slanted parallelepiped, and have found that in some cases it is possible that an optimal speed exists even with a headwind, and that its value is not always v_x.

Lastly, we have considered a vertical cylinder, showing that its behaviour is different from that of a vertical parallelepiped if there is a cross component of the wind velocity.

Our study of the behaviour of solid bodies in the rain has revealed a wide range of situations, and general rules cannot be found. We can say that the presence of a tailwind seems to be a favourable, but not always a necessary condition for the existence of an optimal speed. In some cases, we have found that the value of u_{opt} depends on the drop size, and in other cases it does not.

8.2. Didactic considerations

While the problem seems to be too complex to be proposed as a didactic exercise, some parts of it (e.g. the vertical parallelepiped, or the plane surface) could be suitable for didactic purposes at the undergraduate level. First of all, the problem is familiar to all, and as such stimulates students to apply themselves to find the solution, and also to gaze around with a ‘physical mind’, looking for all the physical phenomena present in everyday life. Moreover, in my opinion it is instructive to see how concepts (like flux, current density, Gauss’s theorem) turn out to be useful in other contexts. At the same time, it is beneficial to think about the fact that concepts are tools, and that they can and should be modified, if necessary, in order to adapt them to our purposes, as we have with the flux concept.

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